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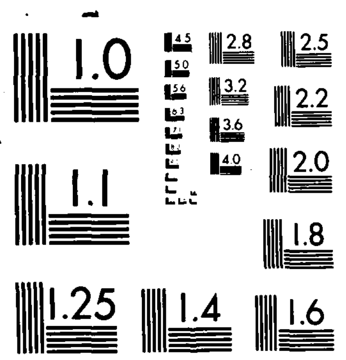
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NSWC TR 86-516

AD-A181 192

A NEW METHOD FOR FINDING PRIME-RICH EQUATIONS OF THE TYPE $I = X^2 - X + C$

BY R. S. SERY

RESEARCH AND TECHNOLOGY DEPARTMENT

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DECEMBER 1986

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A new method has been discovered for finding prime-rich equations of the type $I=x^2-x+c$ where $c=2N-1$ and $N=1,2,3,\ldots$. It is based on an analysis of an array described in NSWC TR 85-120. The array can be defined by stating that all its columns are described by the above equations while simultaneously all its rows are described by the equations $I=x^2+x-r$ where $r=2N-1$ and $N=1,2,3,\ldots$ Primitive cells and primitive cell arrays are derived from the above "special" array by determining and then indicating in table form which columns are

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"empty of integers which are congruent to 0(mod P) where P is prime. The primitive cell array can be thought of as being constructed by the translation in the c and r directions of the primitive cell. Analysis of the superposition of two or more such primitive cell arrays reveals which columns are "empty" of two or more divisors. For example, columns which have no integers congruent to 0(mod 3,5,7 and 11) tend to be prime-rich. By this method 87 prime-rich equations were found where, for x=1 to x_N=40, the density of primes D_c was ≥ 67.5% e.g. for c=21,377 D_{21377} =80%. Fifteen of these equations were richer in primes than Euler's equation, x^2-x+41, when x_N took on the values 2,398 (D_{41}= 50%). 5,000, ...,10,000. For example, for x_N= 10,000, D_{41}=41.49% but D_c (c= 310,691) =45.91%[Abbreviated versions of the special array and the primitive cell of 3 are shown below.

Part of the special array				Primitive cell of 3		
1	5	11	19	-	-	-
3	9	17		3	3	-
7	15	25		-	3	-
13						

FOREWORD

Prime number theory is applicable in disciplines such as encription and artificial intelligence and is therefore of interest to the Navy and DoD. The work reported here is an extension of what has been done previously and described in TR 85-120.

Continued work on this topic was done on the employee's own time and its publication was charged to overhead funds because of it relevance to Navy interests.

The author is grateful to Woodrow Lee for major help in setting up the computer program used, to Schlomo Varsano for advice and help in incorporating his Fast Prime Search program as a subroutine in the program, and Paul Gammell, Lucy Pao, Don Lenko, and Jim Coughlin for their instruction and help in the use of various desk-top and other computers. The author also wishes to thank Larry Hieb for converting the Basic program used with the HP9826 to Fortran, extending the calculation of the D_C values to values of x_N=10,000 and beyond and for corroborating all the previous results obtained in Table 1.

Approved by:

Jack R. Dixon

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Materials Division

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INTRODUCTION

A new method of finding prime-rich equations of the type $I=x^2-x+c$ ($c= 2N-1$, $N=1,2,3,\ldots$) has been devised. The Euler type equation with the highest known density of primes is $I=x^2-x+41$ where the density D_c is defined as $P(x_N)100/x_N$ percent; $P(x_N)$ is the number of primes and x_N is the limiting value for the range $x=1$ to x_N. Unless otherwise stated D_c will be expressed as the percentage for $x_N=40$; for D_{41} this is 100%. By using this method fifteen equations (equivalently fifteen values of c) were found which have higher densities of primes than D_{41} when x_N is set to equal to 10,000. Table 1, shows these equations plus five additional ones whose densities are nearly equal to D_{41}.

PREVIOUS WORK

The method is based on an analysis reported previously.[1] There a special diagonal array of the odd integers was shown to be correlated with equations of the form $I=x^2-x+c$, $c=1$ for the first column of the array, $c=3$ for the second etc. Similarly the rows were described by x^2+x-r where $r=1,3,5,\ldots$ for the rows $N=1,2,3,\ldots$ respectively. An analysis of the rows is a separate study in itself but was not pursued in reference 1 and is not considered

TABLE 1. EQUATIONS x^2-x+c FOR c=41,21377,...... WHERE THE DENSITY, D_c IN PERCENT (100xPRIMES/40) FOR x=1 TO 40 IS > 67.5% AND WHERE D_c IS ALSO GIVEN FOR x_N =100, 200, ETC.

Equations having higher D values then c=41 for x_N= 100000

c	x_N=40	100	200	400	1000	2398	5000	8000	10000
41	100	86	78	67.5	58.1	50	45.22	42.575	41.49
21377	80	75	72	66.75	57.5	51.33..	45.78	43.5	42.55
204791	77.5	57	58.5	56.75	54.7	49.66..	45.68	43.0125	41.82
27941	75	77	74	68.75	59.9	53.25..	48.06	45.625	44.66
55661	75	73	72	69	62.1	54.92..	49.96	46.7875	45.44
22697	75	69	69.5	65	56	49.66..	45.62	43.0125	42.07
72491	75	68	69	67	61	54.37..	49.06	46.2875	45.33
52727	75	66	64	60	54.5	49.87..	45.32.	42.5625	41.55
74147	75	65	64 5	61.75	56.3	50.91..	46.38	43.8125	42.85
41537	70	73	71	68	60.6	53.33..	48.78	45.8	44.80
595937	70	60	62.5	63	60.9	57.96..	53.88	51.05	49.78
47891	67.5	69	67	63.75	57.2	50.66..	46.02	43.4875	42.33
878357	67.5	65	60.5	57.25	53.6	50.625.	48.28.	46.1625	45.10
579431	67.5	64	57	57.5	55.7	53.25..	49.18.	46.9375	45.89
310691	67.5	62	59.5	60	58	53.66..	49.52.	47.225	45.91
Equations with D_c values close to those for c=41									
19421	75	74	70	64	55.8	48.37..	44.12.	41.3625	40.24
31511	67.5	66	64	58.75	54	48.49..	43.76.	41.2	40.32
54407	67.5	66	62	61.25	55.7	50.25..	44.82.	43.325	41.13
41201	67.5	65	63.5	60.5	54.7	48.74..	44.04.	41.55	40.24
221717	67.5	57	56.5	54.5	51.5	47.58..	44.38.	42.25	41.16

here. From the special diagonal array a set of arrays was derived called primitive cell arrays, one for each prime value of c. Each such array has the value c substituted for every integer in the original diagonal array which is $\equiv 0 \pmod{c}$; at each integer, I, not $\equiv 0 \pmod{c}$ the space (corresponding to its position in the original array) is left blank. Reference 1 included tables showing primitive cell arrays for 3,7,11 and a modified one for 5.

Other arrays, not included in the report, were constructed for c=P (P is prime) = 13,17,19,23,.....,41. It was noticed that in each primitive cell array there were "empty" columns (and rows) - by empty is meant columns in which no integers were $\equiv 0 \pmod{P}$. The number of empty columns can be expressed as $N_{ec}=(P-1)/2$ e.g. for P=3 $N_{ec}=1$; for P=13 $N_{e.col.}=6$. It was proved that for the primitive cell arrays of 3 and and 5 the patterns are repeated throughout these arrays no matter how far the columns and rows are extended (for any P the primitive cell pattern consists of a PxP "area" made up P spaces occupied by P's and P^2-P blank spaces). It was assumed, with some supporting evidence, that this holds true for every such array. Table 2 summarizes the equations which describe the number of sets of empty columns for c=P=3,5,7,11,......,37. For the PCA of 3 (PCA will be used instead of primitive cell array for the sake of brevity) the empty columns consist of just one set $N_{ec}=3N+3$, N=0,1,2,3..... . It can be seen that, if one ignores the presence of all other divisors, a column of this set can have a density of 100% e.g. c=41. In contrast those columns which do contain integers which are congruent to 0(mod 3) are described by the two sets 3N+1 and 3N+2, N=0,1,2,3..... . The former set describes the 1^{st}, 4^{th}, 7^{th},..... columns of the cell of 3 for which every third integer of every column is

TABLE 2. EQUATIONS SHOWING THE NUMBER OF SETS OF EMPTY COLUMNS FOR EACH PRIME VALUE OF P FOR THE PRIMITIVE CELL ARRAYS

P	No. of sets	Sets of empty columns per array (N=0,1,2,..)
3	1	3N+3 *
5	2	5N+1, 5N+4
7	3	7N+2, 7N+6, 7N+7
11	5	11N+1, 11N+4, 11N+8, 11N+9, 11N+10
13	6	13N+2, 13N+3, 11N+8, 13N+9, 13N+11, 13N+13
17	8	17N+1, 17N+2, 17N+4, 17N+10, 17N+12,17N+13,17N+14, 17N+17
19	9	19N+2,5,6,11,13,15,16,17,18
23	11	23N+1,4,5,8,10,15,17,18,19,21,23
29	14	29N+1,2,3,4,6,10,13,17,19,20,21,22,25,27
31	15	31N+3,5,7,8,9,14,17,21,22,24,25,27,28,29,30
37	18	37N+3,5,6,7,8,12,17,21,22,23,24,26,29,30,32,34,36,37
-	-	-
-	-	-
P	(P-1)/2	$PN+M_1, M_2, M_3, \ldots\ldots\ldots\ldots, M_{(P-1)/2}$

*Subscripts such as 1,2,3 etc. in $3N_3+3$, $5N_2+1$,..... as shown in Table 3 and elswhere were omitted for the sake of clarity

congruent to 0(mod 3). If just the divisor 3 were to apply to the column it still could only have a maximum D_c of 66 2/3% (65% for x=1 to 40). The latter set applies to the 2^{nd}, 5^{th}, 8^{th},....... columns; here two of every three integers of a column are divisible by 3. The highest density possible is only 33 1/3%; because of the presence of other divisors for both sets, D_c will usually be much lower. The description above was the basis of the criteria formulated in reference 1 which indicated which equations would tend to be prime-rich. The criteria were that c be prime and that x_1, the first value of x used in any column of the diagonal array of Table 1, reference 1, be $\equiv 0 \pmod 3$. The latter criterion still holds; it is just another way of stating that columns 3N+3, N=0,1,2,.... with no integers $\equiv 0 \pmod 3$ are more apt to be prime-rich than columns described by the sets 3N+1 and 3N+2. The second criterion still applies approximately when c is relatively small but as c becomes appreciably larger it is composite for an increasing proportion of prime-rich equations. Therefore if a search for prime-rich equations is limited to values of D > 67.5% only one of every three columns (columns and equations are used interchangeably here) need be examined.

To carry the above analysis one step further one can consider coincidences of empty columns for two or more PCA's. For example there is one set of empty columns for P=3, namely 3N+3 and two for P=5, 5N+1 and 5N+4. It is easy to see that there are empty columns common to the pair 3N+3 and 5N+1 and also to the pair 3N+3 and 5N+4. For the two pairs, for N=0,1,2,..... , 3N+3=3,6,9,12,15,18,21..... and 5N+1=1,6,11,16,21,..... . The equation for columns empty of both the divisors 3 and 5 is given by $N_{3,5}$=15N+6 and

TABLE 3. COINCIDENCES OF EMPTY COLUMNS FOR DIVISORS 3 AND 5; AND 3,5,7; AND 3,5,7,11 FOR DERIVED PRIMITIVE CELL ARRAYS

No. of K family	Equations for "empty" columns for				Families of equations of form x^2-x+c: c=
1	$3N_1+3$	$5N_2+1$	$7N_3+2$	$11N_4+1$	2310N+2201
2	"	"	"	" 4	" " 1151
3	"	"	"	" 8	" " 521
4	"	"	"	" 9	" " 941
5	"	"	"	" 10	" " 1361
6	"	"	$7N_3+6$	$11N_4+1$	" " 221
7	"	"	"	" 4	" " 1481
8	"	"	"	" 8	" " 851
9	"	"	"	" 9	" " 1271
10	"	"	"	" 10	" " 1691
11	"	"	$7N_3+7$	$11N_4+1$	" " 881
12	"	"	"	" 4	" " 2141
13	"	"	"	" 8	" " 1511
14	"	"	"	" 9	" " 1931
15	"	"	"	" 10	" " 41
16	"	$5N_2+4$	$7N_3+2$	$11N_4+1$	" " 1277
17	"	"	"	" 4	" " 227
18	"	"	"	" 8	" " 1907
19	"	"	"	" 9	" " 17
20	"	"	"	" 10	" " 437
21	"	"	$7N_3+6$	$11N_4+1$	" " 1607
22	"	"	"	" 4	" " 557
23	"	"	"	" 8	" " 2237
24	"	"	"	" 9	" " 347
25	"	"	"	" 10	" " 767
26	"	"	$7N_3+7$	$11N_4+1$	" " 2267
27	"	"	"	" 4	" " 1217
28	"	"	"	" 8	" " 587
29	"	"	"	9	" " 1007
30	"	"	"	10	" " 1427

similarly for the second pair we have $N_{3,5}=15N+9$, $N=0,1,2,\ldots$. As $c=2N_{3,5}-1$ we have the equations $x^2-x+30N+K_2$, $N=0,1,2,\ldots$ where $K_2=11$, 17. Thus there two families of equation for coincidences of columns for which there are no integers congruent to 0(mod 3,5). For convenience this is called a (3,5) net. Similarly we can construct the net (3,5,7) for coincidences of empty columns for divisors 3,5 and 7. It is represented by $I=x^2-x+210N+K_6$, $N=0,1,2,\ldots$. This represents six families of equations where $K_6=11,17,41,101,137$, and 167. The next net is that for (3,5,7,11) and its equation is $I=x^2-x+2310N+K_{30}$. K_{30} represents 30 families of equations. These are listed in Table 3. This can be continued to more restrictive nets such as the next in succession, (3,5,7,11,13) whose equation is $x^2-x+30030N+K_{180}$. But to work with this equation and succeeding equations one must evaluate unwieldy numbers of constants. For example for K_{180} there are 180. One member of this family is Euler's equation $I=x^2-x+41$ (this is the 21^{st} column of the original array of Table 1, ref. 1).[1] The next member is $I=x^2-x+30071$ ($N=1$ in the (3,..,13) equation just above); and it would correspond to the 15036^{th} column of the same array. Because of these considerations attention was focused primarily on the family $I=x^2-x+2310N+K_{30}$ (and to a lesser extent, on $x^2-x+210N+K_6$ and $x^2-x+30N+K_6$

CALCULATIONS

Initially x_N was held to the value 40 and N was varied for each of the 30 values of K. For N=0 to 29 this generated 900 equations and D_c was calculated for each. The average value of D_c for all 900 was 47.53%; for 1800

(N=0 to 59) it was about 41.72%. Ultimately the range for N was extended from 0 to 650. In this extended range the largest c value encountered was about 1,500,000. A similar evaluation was made for the (3,5) and (3,5,7) nets. But the largest value of c reached for b h was only about 84,000; for the first net N was ≈ 3000 and for the second it was ≈ 400.

Most of these calculations were accomplished with a Basic program using an HP9826 computer. The program was set up to determine the values x=1 to 40 for each equation and included a Fast Prime Search program as a subroutine. The latter program was developed by S. Varsano[2] and can determine the primality of any integer less than 100,000,000 with double precision. However because the particular computer used was not furnished with adaptation for double precision the values of D_c (see Table 1) could only be determined for x_N values up to about x_N=8,000. The results for x_N=10,000 were obtained by the use of the Laboratory's 720 Cyber computer and all the data shown in Table 1 were confirmed by it. After calculating values of I and indicating whether prime or composite for several hundred equations the program was modified to determine the percentage of I's per column which were prime rather than print out each value and label it prime or composite. As the range of N was increased only those percentages equal to or greater than ≥ 67.5 were printed out. Most of the equations having high D_c values were detected by all three "nets" to the extent that equivalent ranges were covered. However there were exceptions. For example the (3,5) net picked up x^2-x+107 (D_{107}=75%) which the other two did not (and could not by definition for although the column represented by this equation does contain integers divisible by 7 and 17 it contains no integers which are congruent to 0(mod 3,5,11,13,19,23,37,41 and

others)).

Restricting attention to one family of equations alone will not pick up all prime-rich equations but will find most of them. The most complete selection is assured by concentrating on the (3,5) net. However there is a tradeoff. The (3,5) net takes considerably more time (many more equations are processed) than the (3,5,7) or succeeding nets.

RESULTS

Eighty seven equations were found for which was $D_c > 67.5\%$. Of course Euler's equation was one. The next highest D_c value found was that for $I=x^2-x+21377$; $D_{21377}=80\%$. Four other equations shared the next highest values of $D_c=77.5\%$; sixteen had 75% etc. Some percentage values for the case of c=41 are well known; for x=1 to $x_N=100$ $D_{41}=86\%$ and for $x_N=2398$ $D_{41}=50\%$ exactly. In order to compare values of D_c with that for Euler's equation x_N was extended to 100,200,......,10,000. As was pointed out fifteen of the equations of Table 1 were more prime-rich than x^2-x+41 and five were, to a close approximation, almost as prime-rich when $x_N=10,000$. Published results (Ulam et al)[3] state that $D_{41}=47.5\%$ for $I \approx 10^7$, $x_N= 3162$. The results as shown in Table 1, row 1 have extended this to $D_{41}=41.49\%$ for $I \approx 10^8, x_N=10,000$.

Figure 1 is a semilog plot of the density D_c, c=41, 21377, etc. as a function of x_N (x_N= 40,100,....,10,000). The dependent variable D_c is plotted as the abcissa. Only four of the twenty values of c which appear in Table 1 are graphed in Figure 1. As the equations $I=x^2-x+c$ are Diophantine in nature (the I's are integers) the initial parts of the curves where x_N=40,100,...are

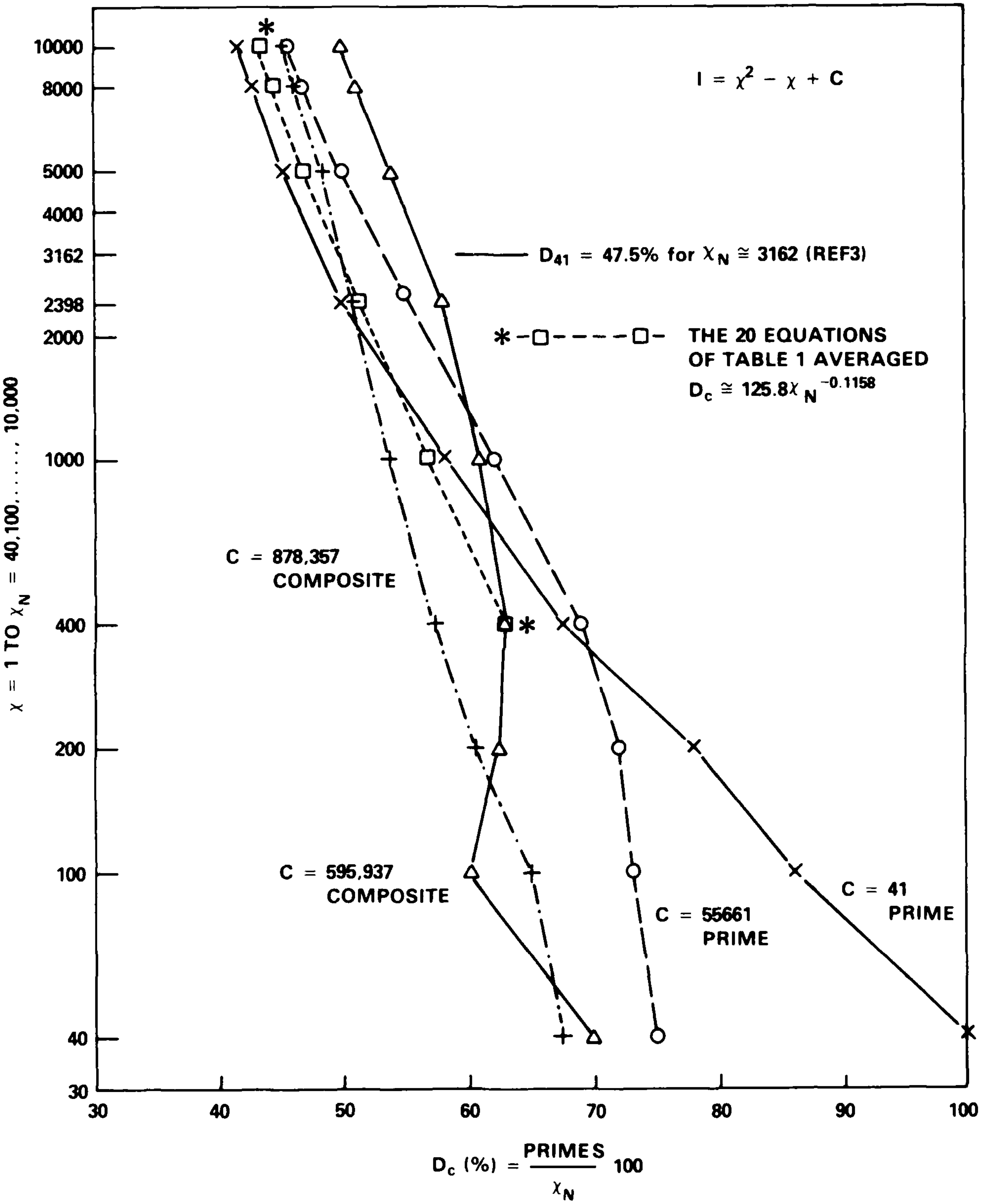

FIGURE 1. SEMILOG PLOTS OF D_c VS χ_N FOR 4 VALUES OF C FROM TABLE 1. THE FIFTH, SHORTER, CURVE IS AN AVERAGE OF ALL 20 VALUES OF THE TABLE.

only very rough approximations of the discontinuous way in which D_c varies with x_N. However if values considerably larger than x_n=10,000 were to be examined the curves although still discontinuous would become much better approximations to smoothly varying curves and would probably have slopes approaching each other in value.

ADDITIONAL OBSERVATIONS

1. C=41 occurs in every family of equations such as $x^2-x+6N+5$ (N=6), $x^2-x+210N+41$ (N=0) and on up to $x^2-x+7,420,738,134,810N+41$ (N=0) the last of which is for the (3,5,7,......,37) net. K_z is the constant for this equation which means that 41 is just one of the 538,876,800 values of z!

2. For the last named family of equations in 1. above the second member is obtained by setting N=1 i.e. $I=x^2-x+7,420,738,134,851$. It is quite possible that for x=1 to 40 no integer of this set is prime. This indicates the impracticality of a direct search for the next equation in the above family for which no integer in the evaluated equation, x^2-x+41, is congruent to 0(mod 3,5,7,........,37). One is more apt to find such an equation incidently by using nets such as (3,5), (3,5,7) or (3,5,7,11). $x^2-x+107$ was found in this manner (see page). Similarly $x^2-x+21377$ was found to be the 10^{th} member of the family $x^2-x+2310N+587$ (N=9): it is also the first member of $x^2-x+200,560,490,120N+K_z$ which is the equation for the (3,5,7,......,31) net (that is for N=0, K=21377 and z represents 29,937,600 constants).

3. Two rules for calculation were found with regard to primitive cells (by primitive cell is meant the area of the primitive cell itself comprised of the P x P spaces and not the whole primitive cell array). These rules, though not proved, work. The first is that the number of empty columns in a cell is equal to (P-1)/2. The second is that the number of coincidences of empty columns is given by $(P_N-1)/2$ x......x $(P_2-1)/2$ x $(P_1-1)/2$. For example for $P_N=11$ $N_{ec}=(11-1)/2 \times (7-1)/2 \times (5-1)/2 \times (3-1)/2=30$. This gives the number of values of the constant K required to be evaluated in $x^2-x+2310N+K_{30}$ (see the last column of Table 3).

4.The general equation for any net is $x^2-x+N \prod_{P=2}^{P=P} P+K_{(p)}$; $K_{(p)}$ is the number but not the values of the constants to be evaluated and is given by $(P_p-1)/2x,.....,x(P_3-1)/2 \times (P_2-1)/2$. Special cases are: if P=2 then $K_{(1)}= -1$ and if P=3 then $K(1)=1$ (expressions P_N in 3. and P_p here are interchangable). For P=13 $I=x^2-x+N(13x11x7x5x3x2)+K_{(P)}=x^2-x+30030N+K_{180}$ where $K_{(p)}$ is equal to $(13-1)/2 \times (11-1)/2 \times (7-1)/2 \times (5-1)/2 \times (3-1)/2=180$ i.e. $K_{(p)}=K_{(13)}=K_{180}$

5. It is worth noting that for all 87 equations having values of D_c > 67.5%, 20 of which are included in Table 1, the last digit for every constant c is either a 1 or a 7. This is a consequence of how the nets are used to find prime-rich equations. For the (3) net i.e. for p=3 in 4 above, $I=x^2-x+6N+5$, N=0,1,2.... can have any odd integer as a unit digit. However in order to more readily determine which column (equation) would be prime-rich it was necessary to go to the (3,5) and succeeding nets. For the (3,5) net

$I=x^2-x+30N+11,17$. For the two families represented here, where $N=0,1,2......$, every c value will be an integer whose units digit is 1 or 7. As all succeeding nets involve multiples of the repeat interval for coincident empty columns for the (3,5) net any c value for all nets beyond the (3,5) net will also have a 1 or 7 for the units digit.

CONCLUSIONS

It has been shown that there are only six equations of the type $I=x^2-x+c$, $x=1,2,3,....$(x^2+x+c, $x=0,1,2,.....$ is an equivalent form) for which all values of x from 1 to c-1 yield prime values of I.[4] They are c=2,3,5,11,17 and 41. c=1 could be a seventh but the solution $I=f(x)=1$ is not prime. Of the above Euler's has been the most prime-rich equation known. However the above constraint does not rule out the possibility that other equations of this form can be just as rich or more so. In fact it has been shown above (Table 1) that there are such equations and that there is a viable method of finding them. Work is being completed on an analogous set of at least 16 equations of the related type $I=x^2+x-r$ e.g. for r=501,229 and $x_N=10,000$ $D_r=49.25\%$. This equation is a "row" equation of the special array described in reference 1 in contrast to the "column" equations discussed in this report.

It should be feasible to examine c values greater than 1.5×10^6. Between c=858,707 of Table 1 and c= 1.5×10^6 the only c value found for which D_c was equal to 67.5% was c=1,275,707; D_c for $x_N=10,000$ was about 37.5%. It is anticipated that there will be few if any D_c's J 67.5% between 1.5×10^6, the highest c value examined, and 3.0×10^6.

It should also be worthwhile to extend the range of x_N beyond 10,000 for all values of c in Table 3. Using only the five viable points available

for D_{41} for x_N=1,000 to 10,000 the following empirical equation was found: $D_{41}=141.9(x_N^{-0.134})$. The value of D_{41} for x_N=11,000 has been found to be 40.96..%*; that is 4506 out of 11,000 I's (integers) are prime.[5] Substitution of x_N= 11,000 in the above equation yields D_{41}=40.78 (or 4486 primes out of 11,000 I's), which differs from the true value by less than 0.5%. Substitution of x_N= 100,000 in the same equation yields ≈ 30%. However, how valid this figure may be depends on obtaining more points in the relation $D_c=f(x_N)$ and finding a better fit to the plotted curve.

Two other items of possible interest related to this report which may be worth looking into are to examine those columns and rows (equations) which are very prime-poor and to follow-up a number of equations which were found to be prime-rich but fell just short of the arbitrarily set D_c criterion of being > 67.5%. One such equation was for c=1,384,907. The D_c values for x_N = 40, 2,398 and 8,000 were 65, 42.66.. and 38.725% respectively and the projected density for x_N=10,000 is about 36%.

* The Cyber 720 gave 40.97....%; for x_N=12,000 it gave 40.35%.

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